10.3. By using eq. (9.3), we can easily show that

$$\alpha^n u[-n-n_0] \overset{z}{\longleftrightarrow} \frac{-z^{-n_0}}{1-\alpha z^{-1}}, \qquad |z| < |\alpha|.$$

We then obtain

$$X(z) = \frac{1}{1+z^{-1}} + \frac{-z^{-n_0-1}}{1-\alpha z^{-1}}, \qquad 1 < |z| < |\alpha|.$$

Therefore, $|\alpha|$ has to be 2. n_0 can take on any value.

10.4. Using eq. (9.3), we have

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{0} (\frac{1}{3})^n \cos(\frac{\pi}{4}n) z^{-n} \\ &= (1/2) \sum_{n=-\infty}^{0} (\frac{1}{3})^n e^{j\pi n/4} z^{-n} + (1/2) \sum_{n=-\infty}^{0} (\frac{1}{3})^n e^{-j\pi n/4} z^{-n} \\ &= (1/2) \sum_{n=0}^{\infty} (\frac{1}{3})^{-n} e^{-j\pi n/4} z^{n} + (1/2) \sum_{n=0}^{\infty} (\frac{1}{3})^{-n} e^{j\pi n/4} z^{n} \\ &= (1/2)\frac{1}{1-3e^{-j\pi/4}z} + (1/2)\frac{1}{1-3e^{j\pi/4}z}, \qquad |z| < \frac{1}{3} \end{aligned}$$

The poles are at $z = \frac{1}{3}e^{j\pi/4}$ and $z = \frac{1}{3}e^{-j\pi/4}$.

10.5. **(a)** The given z-transform may be written as

$$X(z) = \frac{z-\frac{1}{2}}{(z-\frac{1}{3})(z-\frac{1}{4})}.$$

Clearly, $X(z)$ has a zero at $z = \frac{1}{2}$. Since in $X(z)$ the order of the denominator polynomial exceeds the order of the numerator polynomial by 1, $X(z)$ has a zero at ∞. Therefore, $X(z)$ has one zero in the finite z-plane and one zero at infinity.

(b) The given z-transform may be written as

$$X(z) = \frac{(z-1)(z-2)}{(z-3)(z-4)}.$$

Clearly, $X(z)$ has zeros at $z = 1$ and $z = 2$. Since in $X(z)$, the orders of the numerator and denominator polynomials are identical, $X(z)$ has no zeros at infinity. Therefore, $X(z)$ has two zeros in the finite z-plane and no zeros at infinity.

(c) The given z-transform may be written as

$$X(z) = \frac{(z-1)}{z(z-\frac{1}{4})(z+\frac{1}{4})}.$$

Clearly, $X(z)$ has a zero at $z = 1$. Since in $X(z)$ the order of the denominator polynomial exceeds the order of the numerator polynomial by 2, $X(z)$ has two zeros at ∞. Therefore, $X(z)$ has one zero in the finite z-plane and two zeros at infinity.

(c) From Section 10.4, we know that the magnitude of the Fourier transform ma expressed as

$$|H_3 e^{j\omega})| = \frac{(\text{Length of } \vec{v_1})^2}{(\text{Length of } \vec{v_2})(\text{Length of } \vec{v_3})},$$

where $\vec{v_1}$, $\vec{v_1}$, and $\vec{v_3}$ are as shown in the figure above. Clearly, for small value ω (ω near zero), and for values of ω near π the numerator of the right-hand sid the above equation is almost the same as the denominator. But when $|\omega|$ is $\pi/2$, the numerator of the right-hand side of the above equation is much larger t the denominator. Therefore, $H_1(e^{j\omega})$ is large near $\omega = \pi/2$. Therefore, $H_1(e^{j\omega}$ approximately bandpass.

10.13. (a) The signal $g[n]$ is

$$g[n] = \delta[n] - \delta[n-6].$$

Using the definition of the z-transform in eq. (10.3), we obtain

$$G(z) = 1 - z^{-6}, \qquad |z| > 0.$$

(b) From Table 10.1, we have

$$x[n] = \sum_{k=-\infty}^{n} g[k] \overset{Z}{\longleftrightarrow} X(z) = \frac{1}{1-z^{-1}} G(z), \qquad \text{At least} |z| > 1.$$

Therefore,

$$X(z) = \frac{1-z^{-6}}{1-z^{-1}}, \qquad |z| > 0.$$

The ROC is $|z| > 0$ because $x[n]$ is a finite-length signal.

10.14. (a) We know that $x[n] * x[n]$ will be triangular signal whose first non-zero value occurs $n = 0$. Furthermore, we also know that $x[n] * x[n - n_0]$ has its first nonzero value $n = n_0$. Therefore, $n_0 = 2$.

(b) From Problem 10.13 we have

$$X(z) = \frac{1-z^{-6}}{1-z^{-1}}, \qquad |z| > 0.$$

Using the shift property,

$$x[n-2] \overset{Z}{\longleftrightarrow} z^{-2}\frac{1-z^{-6}}{1-z^{-1}}, \qquad |z| > 0.$$

Using the convolution property,

$$g[n] = x[n] * x[n-1] \overset{Z}{\longleftrightarrow} z^{-2}\left(\frac{1-z^{-6}}{1-z^{-1}}\right)^2, \qquad |z| > 0.$$

Since

$$\lim_{z\to\infty} G(z) = 0 = g[0],$$

$G(z)$ does satisfy the initial value theorem.

10.15. Taking the z-transform of $y[n]$, we have

$$Y(z) = \frac{1}{1 - \frac{1}{9}z^{-1}}, \qquad |z| > \frac{1}{9}.$$

Now from Table 10.1, we have

$$y_1[n] = y_{(2)}[n] = \begin{cases} y[r], & ,n = 2r \\ 0, & ,n \neq 2r \end{cases} \quad \stackrel{z}{\longleftrightarrow} Y_1(z) = Y(z^2), \qquad |z| > \frac{1}{3}.$$

Therefore,

$$y_1[0] = 1,\ y_1[1] = 0,\ y_1[2] = \frac{1}{9},\ y_1[3] = 0,\ y_1[4] = \frac{1}{81}, \cdots$$

This may be written as

$$y_1[n] = \frac{1}{2}\left[\left(\frac{1}{3}\right)^n u[n] + (-1)^n \left(\frac{1}{3}\right)^n u[n]\right].$$

If we now choose $x[n]$ to be $\frac{1}{2}\left[\left(\frac{1}{3}\right)^n u[n]\right]$, then

$$Y_1(z) = Y(z^2) = (1/2)[X(z) + X(-z)], \qquad |z| > \frac{1}{3}.$$

Furthermore, since $X(z)$ has only one pole and one zero, this choice of $x[n]$ satisfies both the given conditions.

We may also choose $x[n]$ to be $\frac{1}{2}\left[(-1)^n \left(\frac{1}{3}\right)^n u[n]\right]$. This would still satisfy both given conditions.

10.16. For a system to be both causal and stable, the corresponding z-transform must not have any poles outside the unit circle.

(a) The given z-transform has a pole at infinity. Therefore, it is **not causal.**

(b) The poles of this z-transform are at $z = \frac{1}{4}$ and $z = -\frac{3}{4}$. Therefore, it is **causal.**

(c) This z-transform has a pole at $-\frac{4}{3}$. Therefore, it is **not causal.**

10.17. **(a)** Since $\lim_{z\to\infty} = 1$, $H(z)$ has no poles at infinity. Furthermore, since $h[n]$ is given to be right-sided, $h[n]$ has to be causal.

(b) Since $h[n]$ is causal, the numerator and denominator polynomials of $H(z)$ have the same order. Since $H(z)$ is given to have two zeros, we may conclude that it also has two poles.

Since $h[n]$ is real, the poles must occur in conjugate pairs. Also, it is given that one of the poles lies on the circle defined by $|z| = \frac{3}{4}$. Therefore, the other pole also lies on the same circle.

Clearly, the ROC for $H(z)$ will be of the form $|z| > \frac{3}{4}$. and will include the unit circle. Therefore, we may conclude that the system is stable.

(b) $X(z)$ may be rewritten as

$$X(z) = \frac{z^2}{(z-\frac{1}{2})(z-1)}.$$

Using partial fraction expansion, we may rewrite this as

$$\begin{aligned} X(z) &= 2z^2\left[-\frac{1}{z-\frac{1}{2}}+\frac{1}{z-1}\right] \\ &= 2z\left[-\frac{z}{z-\frac{1}{2}}+\frac{z}{z-1}\right] \end{aligned}$$

If $x[n]$ is right-sided, then the ROC for this signal is $|z| > 1$. Using this fact, we may find the inverse z-transform of the term within square brackets above to be $y[n] = -(1/2)^n u[n] + u[n]$. Note that $X(z) = 2zX(z)$. Therefore, $x[n] = 2y[n+1]$. This gives

$$x[n] = -2\left(\frac{1}{2}\right)^{n+1} u[n+1] + 2u[n+1].$$

Noting that $x[-1] = 0$, we may rewrite this as

$$x[n] = -\left(\frac{1}{2}\right)^{n} u[n] + 2u[n].$$

This is the answer that we obtained in part (a).

10.26. (a) From part (b) of the previous problem,

$$X(z) = \frac{z^2}{(z-\frac{1}{2})(z-1)}.$$

(b) From part (b) of the previous problem,

$$X(z) = 2z\left[-\frac{z}{z-\frac{1}{2}}+\frac{z}{z-1}\right].$$

(c) If $x[n]$ is left-sided, then the ROC for this signal is $|z| < 1/2$. Using this fact, we may find the inverse z-transform of the term within square brackets above to be $y[n] = (1/2)^n u[-n-1] - u[-n-1]$. Note that $X(z) = 2zX(z)$. Therefore, $x[n] = 2y[n+1]$. This gives

$$x[n] = 2\left(\frac{1}{2}\right)^{n+1} u[-n-2] - 2u[-n-2].$$

10.27. We perform long-division on $X(z)$ so as to obtain a right-sided sequence. This gives us

$$X(z) = z^3 + 4z^2 + 5z + \cdots.$$

Therefore, comparing this with eq. (10.3) we get

$$x[-3] = 1, \qquad x[-2] = 4, \qquad x[-1] = 5,$$

and $x[n] = 0$ for $n < -3$.

Then,

$$x[n] = 0, \qquad \text{for } n < -1,$$
$$x[-1] = 2/3 - 2/3 = 0,$$
$$x[n] = 0, \qquad \text{for } n > 0.$$

It follows that $x[n] = \delta[n]$. It can similarly be shown that $h_2[n]$ and $h_3[n]$ satisfy the difference equation.

10.36. Taking the z-transform of both sides of the given difference equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} - \frac{10}{3} + z} = \frac{z^{-1}}{1 - \frac{10}{3}z^{-1} + z^{-2}}.$$

The partial fraction expansion of $H(z)$ is

$$H(z) = -\frac{3/8}{1 - \frac{1}{3}z^{-1}} + \frac{3/8}{1 - 3z^{-1}}.$$

Since $H(z)$ corresponds to a stable system, the ROC has to be $(1/3) < |z| < 3$. Therefore,

$$h[n] = -\frac{3}{8}\left(\frac{1}{3}\right)^n u[n] - \frac{3}{8}(3)^n u[-n-1].$$

10.37. (a) The block-diagram may be redrawn as shown in part (a) of the figure below. This may be treated as a cascade of the two systems shown within the dotted lines in Figure S10.37. These two systems may be interchanged as shown in part (b) of the figure Figure S10.37 without changing the system function of the overall system. From the figure below, it is clear that

$$y[n] = x[n] + \frac{9}{8}x[n-1] - \frac{1}{3}y[n-1] + \frac{2}{9}y[n-2].$$

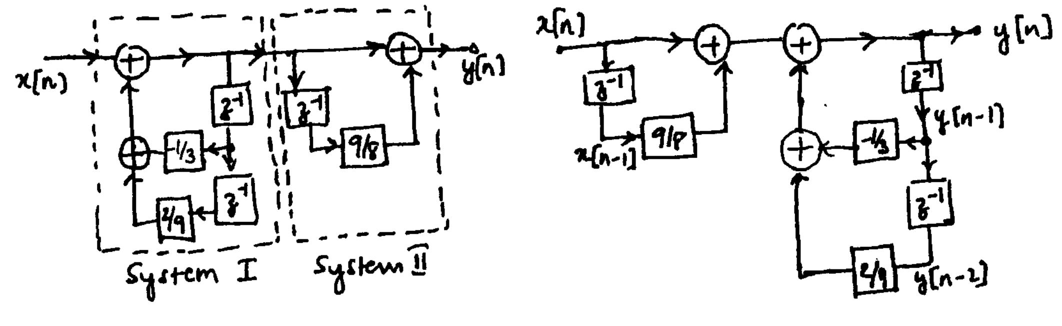

Figure S10.37

(b) Taking the z-transform of the above difference equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{9}{8}z^{-1}}{1+\frac{1}{3}z^{-1}-\frac{2}{9}z^{-2}} = \frac{1+\frac{9}{8}z^{-1}}{(1+\frac{2}{3}z^{-1})(1-\frac{1}{3}z^{-1})}.$$

$H(z)$ has poles at $z = 1/3$ and $z = -2/3$. Since the system is causal, the ROC has to be $|z| > 2/3$. The ROC includes the unit circle and hence the system is stable.

10.38. (a) $e_1[n] = f_1[n]$.

(b) $e_2[n] = f_2[n]$.

(c) Using the results of parts (a) and (b), we may redraw the block-diagram as shown in Figure S10.38.

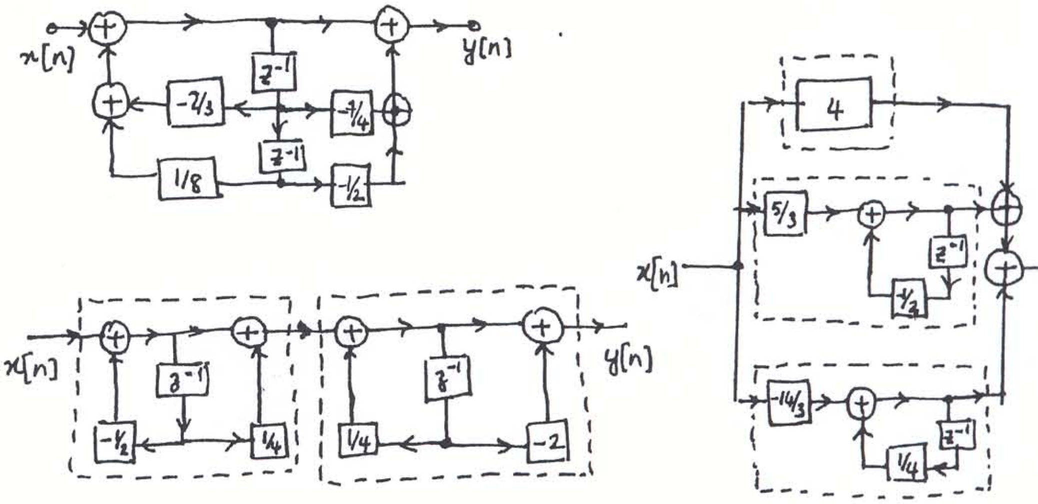

Figure S10.38

(d) Using the approach shown in the examples in the textbook we may draw the block-diagram of $H_1(z) = [1+(1/4)z^{-1}]/[1+(1/2)z^{-1}]$ and $H_2(z) = [1-2z^{-1}]/[1-(1/4)z^{-1}]$ as shown in the dotted boxes in the figure below. $H(z)$ is the cascade of these two systems.

(e) Using the approach shown in the examples shown in the textbook, we may draw the block-diagram of $H_1(z) = 4$, $H_2(z) = [5/3]/[1+(1/2)z^{-1}]$ and $H_3(z) = [-14/3]/[1-(1/4)z^{-1}]$ as shown in the dotted boxes in the figure below. $H(z)$ is the parallel combination of $H_1(z)$, $H_z(z)$, and $H_3(z)$.

10.39. (a) The direct form block diagram may be drawn as shown in part (a-i) of Figure S10.39 by noting that

$$H_1(z) = \frac{1}{1-\frac{5}{3}z^{-1}-\frac{11}{36}z^{-2}-\frac{5}{18}z^{-3}+\frac{1}{36}z^{-4}}.$$

The cascade block-diagram is as shown in part (a-ii) of Figure S10.39.

10.59. **(a)** From Figure S10.59, we have

$$W_1(z) = X(z) - \frac{k}{3}z^{-1}W_1(z) \quad \Rightarrow \quad W_1(z) = X(z)\frac{1}{1+\frac{k}{3}z^{-1}}.$$

Also,

$$W_2(z) = -\frac{k}{4}z^{-1}W_1(z) = -X(z)\frac{\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}.$$

Therefore, $Y(z) = W_1(z) + W_2(z)$ will be

$$Y(z) = X(z)\frac{1}{1+\frac{k}{3}z^{-1}} - X(z)\frac{\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}.$$

Finally,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}.$$

Since $H(z)$ corresponds to a causal filter, the ROC will be $|z| > |k|/3$.

(b) For the system to be stable, the ROC of $H(z)$ must include the unit circle. This is possible only if $|k|/3 < 1$. This implies that $|k|$ has to be less than 3.

(c) If $k = 1$, then

$$H(z) = \frac{1-\frac{1}{4}z^{-1}}{1+\frac{1}{3}z^{-1}}.$$

The response to $x[n] = (2/3)^n$ will be of the form

$$y[n] = x[n]H(2/3) = \frac{5}{12}(2/3)^n.$$

10.60. The unilateral z-transform of $y[n] = x[n+1]$ is

$$\begin{aligned}
\mathcal{Y}(z) &= \sum_{n=0}^{\infty} y[n]z^{-n} \\
&= y[0] + y[1]z^{-1} + y[2]z^{-2} + \cdots \\
&= x[1] + x[2]z^{-1} + x[3]z^{-2} + \cdots \\
&= z\{x[0] + x[1]z^{-1} + x[2]z^{-2} + x[3]z^{-3} + \cdots\} - zx[0] \\
&= z\mathcal{X}(z) - zx[0].
\end{aligned}$$